Resource Allocation Problems on Bipartite Graphs with Assignment Restrictions

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1 Introduction

Efficient use of resources is a fundamental and longstanding problem. Recently, energy saving has been a global issue, and alternative power sources other than thermal power, such as solar or wind, has become popular. Future power networks will have multiple, different kind of power sources, and it will be of great importance to use them in a appropriate and efficient way.

[1] proposes a future power architecture with ICT-based intelligent power control. In this architecture, when a power consuming device is connected to a network, the device will send a request. Then the network computes the best matched power source for the device, and the matched source will begin to supply. In this on-demand power network architecture, it is highly important to use an efficient algorithm to decide the best matched source.

In this paper, we formulate the problem as variants of resource allocation problems, and analyze their computational complexity.

2 Models and Results

In this paper, we extend the standard resource allocation problem in the viewpoint of the future power networks described above. Especially, we focus one specific model, which can be named as a splittable resource allocation problem with assignment restrictions.

Generally, the standard resource allocation problem is described as follows; there are a set of requests (from consuming devices) and another set of sources (power sources). A request has two parameters, profit and weight, while a source has one, capacity. The task of an algorithm is to find the allocation of requests that maximizes the total profit.

We consider two additional factors. The one is assignment restrictions, which means that each request has a subset of sources, and it can be supplied only from sources in the subset. The other is splittable or unsplittable; in a splittable model, multiple sources can serve one request cooperatively.

2.1 Related Results

When there is no assignment restrictions and requests are splittable, the problem is NP-hard since it includes the knapsack problem, which is NP-hard [5]. The problem with no assignment restriction and unsplittable requests is totally relevant to the multiple knapsack problem, which is strongly NP-hard even when the number of sources are restricted to 2 [2], while there is a PTAS (Polynomial Time Approximation Scheme) [2].

In the case when requests have assignment restrictions and are unsplittable, the problem is same as the knapsack problem with assignment restrictions, known to be strongly NP-hard [3]. There is a 2-approximation algorithm when each item’s profit equals its weight [3].

2.2 Our Result

We consider the resource allocation problem with assignment restrictions and splittable requests, and prove that it is hard to be approximated within a factor of \(n^{1-o(1)}\), even when we restrict profits of each request to be 1. Proof is done by reduction from the independent set problem, which is also known to be hard to be approximate within \(n^{1-o(1)}\). Details are omitted due to the space limitation.

Table 1 shows our classification of the models and results.
Table 1: Model classification and results.

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<tr>
<th></th>
<th>Splittable</th>
<th>Not splittable</th>
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<tbody>
<tr>
<td>No assignment restrictions</td>
<td>NP-hard[5]</td>
<td>Strongly NP-hard</td>
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<tr>
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<td></td>
<td>PTAS[2]</td>
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<tr>
<td>With assignment restrictions</td>
<td>Hard to be approximated within a ratio of $n^{1-o(1)}$ [This]</td>
<td>Strongly NP-hard (when profit = weight, 2-approximation [3])</td>
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3 Future Works

One of the future works is to consider some models with some realistic assumptions, such as an online model in which algorithms can never know about future requests in advance.

References


